The University of Nottingham

DEPARTMENT OF MECHANICAL, MATERIALS AND MANUFACTURING ENGINEERING

A LEVEL 2 MODULE, SPRING SEMESTER 2013-2014

MECHANICS OF SOLIDS 3

Time allowed TWO Hours

Candidates may complete the front cover of their answer book and sign their desk card but must NOT write anything else until the start of the examination period is announced

Answer any FOUR questions

Only silent, self contained calculators with a Single-Line Display, or Dual-Line Display are permitted in this examination.

Dictionaries are not allowed with one exception. Those whose first language is not English may use a standard translation dictionary to translate between that language and English provided that neither language is the subject of this examination. Subject specific translation dictionaries are not permitted.

No electronic devices capable of storing and retrieving text, including electronic dictionaries, may be used.

DO NOT turn examination paper over until instructed to do so

If students answer more than the required number of questions only the required number will be marked. If during the exam students attempt additional questions they should clearly indicate on their script which solutions they want to be marked - simply putting a line through solutions that should be disregarded is recommended.

ADDITIONAL MATERIAL: None

INFORMATION FOR INVIGILATORS:

Question papers should be collected in at the end of the exam – do not allow candidates to take copies from the exam room.

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1. Figure Q1 shows the cross-section of a beam.

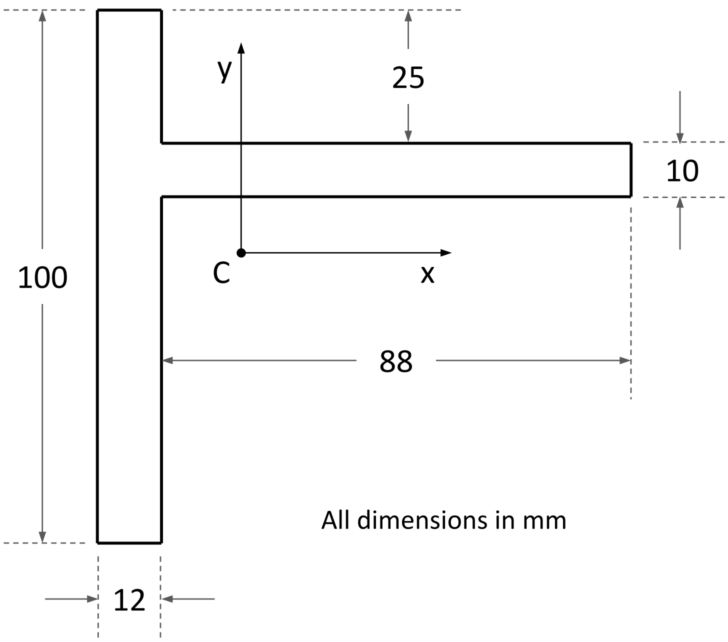

Figure Q1

Determine:

(a) The position of the Centroid of Area, C.

[6 marks]

(b) The Principal 2nd Moments of Area.

[13 marks]

(c) The orientation of the Principal Axes with respect to the x-y co-ordinate system (show on a sketch of the cross-section).

[6 marks]

2.

(a) Derive an expression for the strain energy in a uniform section beam of length, L, with Young's Modulus, E, second moment of area, I, subjected to a uniform bending moment, M, along its length.

[5 marks]

(b) The continuous beam shown in Figure Q2 is made up of a straight vertical section of length, L, of 400mm, which is built in to the ground at one end and attached to a quarter circular section of 200mm radius, R, at the other end. A point load, P, of 1.25kN, is applied at the tip of the quarter circular section as shown in the figure. The beam has a solid circular cross-section of 40mm diameter, D. The beam is made of steel with a Young's Modulus, E, of 210GPa.

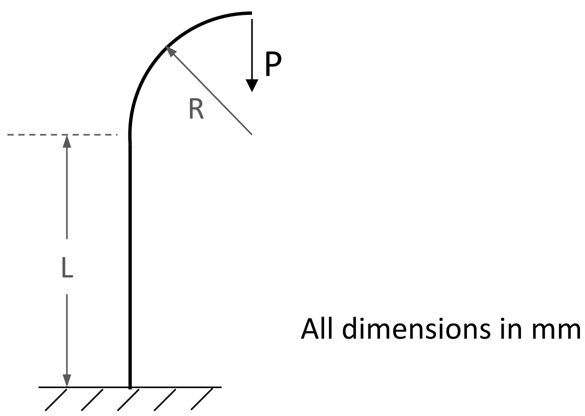

Figure Q2

Determine the vertical and horizontal deflections at the tip of the beam (at the position of the applied load, P).

[20 marks]

The following trigonometric identities may be useful when solving for the deflections:

$sin^2\phi + cos^2\phi = 1$ $cos2\phi = cos^2\phi - sin^2\phi$ $sin2\phi = 2sin\phi cos\phi$

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3. An aluminium flywheel is shrink-fitted to a steel shaft. The steel shaft is 50mm diameter and the flywheel has an outer diameter of 200mm. Prior to shrink-fitting, the inside diameter of the flywheel is 0.5 mm smaller than the diameter of the shaft.

(a) Find the radial pressure between the shaft and the flywheel.

[20 marks]

(b) Sketch the hoop stress and radial stress distribution in the flywheel.

[5 marks]

For steel, $E_{steel} = 200 \times 10^9$ N/m^2 , $\nu_{steel} = 0.3$.

For aluminium, $E_{aluminium} = 70 \times 10^9$ N/m^2 , $\nu_{aluminium} = 0.33$.

4.

(a) For a steel strut of rectangular cross section 25mm x 50mm, length 1m, determine the buckling collapse load for the following end conditions:

(i) Pinned - pinned
(ii) Fixed – fixed
(iii) Fixed – pinned (free)
(iv) Fixed - pinned (forced to translate only along axis of strut so that y displacement at end is zero)

[10 marks]

(b) A mass is supported by a steel framework of 4 struts. The struts are arranged so that at the upper end of each strut, the centre-lines of the struts lie, equally spaced, on a circle of 100mm diameter. At the lower end of each strut, the centre-lines of the struts lie, equally spaced, on a circle of 1m diameter. The height of the framework (i.e. the distance between the planes of these circles) is 1500mm. Each strut has a solid circular cross-section.

(i) Determine the diameter of the struts (assume pinned at both ends) which will give an equal load for failure by buckling and failure by plastic collapse.

[15 marks]

Assume $E_{steel} = 209 \times 10^9$ N/m^2, yield stress of steel = 250MPa

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5. The section shown in Figure Q5 carries a vertical shear force, S = 50kN acting down the vertical centre line.

(a) Calculate the second moment of area of the section about the neutral axis (N.A.).

[7 marks]

(b) Determine the vertical shear stress at points A, B, C, D and E.

[13 marks]

(c) Sketch the vertical shear stress distribution in the section.

[5 marks]

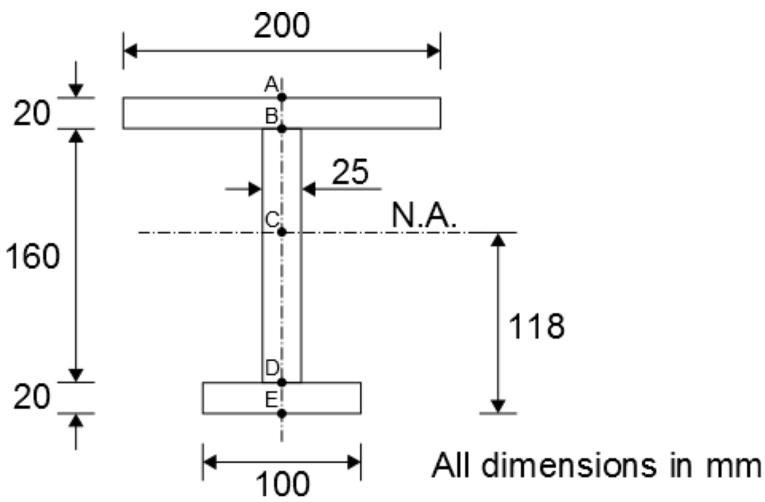

Figure Q5

6.

(a) Finite Element Analysis (FEA) is to be carried out of the uniaxially loaded notched bar in Figure Q6a. Given the geometry and loading conditions, describe two simplifications that can be made when modelling the bar and sketch the geometry, loading and boundary conditions of the model.

[7 marks]

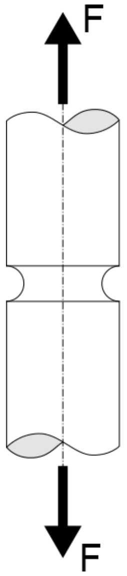

Figure Q6a

(b) The pin-jointed framework is subjected to an external load as shown in Figure Q6b. If each member has a length *L*, cross sectional area *A* and modulus of elasticity *E:*

(i) construct the stiffness matrix of the structure.

[10 marks]

(ii) determine expressions for the horizontal and vertical displacements at point B.

[3 marks]

(iii) determine expressions for the reaction forces at points A and C.

[5 marks]

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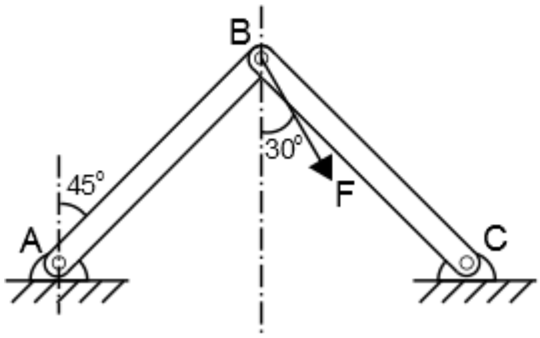

Figure Q6b

The stiffness matrix of a truss element is:

$$[k_e] = \left(\frac{AE}{L}\right)\begin{bmatrix} \cos^2\theta & \cos\theta\sin\theta & -\cos^2\theta & -\cos\theta\sin\theta \\ \cos\theta\sin\theta & \sin^2\theta & -\cos\theta\sin\theta & -\sin^2\theta \\ -\cos^2\theta & -\cos\theta\sin\theta & \cos^2\theta & \cos\theta\sin\theta \\ -\cos\theta\sin\theta & -\sin^2\theta & \cos\theta\sin\theta & \sin^2\theta \end{bmatrix}$$

End